Growth Modelling — a (Re)view

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Growth models are vitally important for forest management planning. Forecasting the growth and yield of individual stands is a prerequisite for planning the management of forests at any level. Therefore, managers need to have an appreciation of the various growth modelling techniques and their limitations.

More than a review of growth models, this is an examination of basic principles as I see them. An exhaustive review would take far too much space, would probably be of more interest to the specialist, and has already been done to some extent in some of the publications cited below. Instead, I have aimed at a contribution toward a better understanding of the relevant literature.

The focus is on modelling techniques appropriate for even-aged stands. The references selected to illustrate the various methods are a more or less random sample. However, following tradition, only publications in English are included, and the sample is heavily biased toward the author's own work.

New Zealand growth models have been reviewed by Goulding, 1986.

Foundations

"Static" (Alder 1980) growth models attempt to predict directly the course over time of the quantities of interest (volumes, mean diameter). Examples of these are the Forestry Commission Management Tables (Johnston and Bradley 1963) and the South Australian Yield Tables (Lewis et al 1976). This approach can give good results for unthinned stands, or for stands subject to a limited range of standardized treatments for which long-term experimental data are available.

Dynamic models are needed for forecasting over a wider range of tending regimes (initial spacing, various thinning and pruning sequences and intensities). Instead of modelling directly the course of values over time, these models predict rates of change under various conditions. The trajectories over time are then obtained by adding or integrating these rates.

Growth modelling may be clarified through the use of some very simple basic concepts that have been fundamental in other disciplines for a long time. Essentially, the evolution over time of any system can be modelled by specifying:

1. An adequate description of the system at any point in time (the "state" of the system).

2. The rate of change of state as a function of the current state and of the current value of any external control variables (a "local transition function").

The state must be such that, to a sufficient degree of approximation: (a) future states are determined by the current state and future actions, and (b) any characteristics of interest can be derived from the state. The state is often described by a fixed number of variables (state variables, forming a state vector). It can also include more complicated mathematical objects such as infinite sequences and functions. Transition functions are typically given as a system of differential or difference equations, describing the rate of change of each of the state variables. Alternatively, they might be specified graphically, or through a more or less involved computational algorithm.

Let us illustrate these principles in the context of growth modelling. First, control variables usually need not be included in the transition function. Silvicultural treatments (e.g. thinnings) normally happen at discrete points in time, causing an instantaneous change of state.

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We can then model the changes of state in between treatments as a function of the current state only, without any control variables.

Consider the mean top height, H, as a state description for a stand. This satisfies condition (a) above, since the rate of change of H (height increment) can be modelled adequately as a function of the current H: $\Delta H = f(H)$, or $\mathrm{d}H/\mathrm{d}t = g(H)$. The course of H over time can be obtained by accummulating or integrating the increments, starting from a given initial height. If we are interested in the volume per hectare, H is not a good state description according to condition (b): the volume depends also on the basal area in addition to H. The model may, however, be adequate for other purposes, e. g. for site quality classification.

Consider now the total volume per hectare, as a state description. If we are interested in forecasting this quantity, it obviously satisfies condition (b) (although it would probably not be sufficient if we were also interested in merchantable volumes). However, condition (a) fails because, over a wide range of treatments, the volume increment would be different for stands having the same volume but very different heights and/or stockings.

It appears that a one-dimensional state is inadequate for growth modelling. Consider then the state being described by three variables: basal area (G), stems per hectare (N), and top height (H). That is, the state is the three-dimensional vector $\mathbf{x} = (G, N, H)$. In many instances this satisfies condition (a), the changes in $\mathbf{x}$ for a wide variety of treatments being well approximated by a function of $\mathbf{x}$, $\Delta\mathbf{x} = \mathbf{f}(\mathbf{x})$, that is, by a system of three equations: $\Delta G = f_1(G, N, H)$, $\Delta N = f_2(G, N, H)$, $\Delta H = f_3(G, N, H)$. Often this state vector is satisfactory also according to (b), with volumes of various products, values, and size distribution parameters being estimated by regression on G, N and H. Notice that any one-to-one transformation of this vector would serve as well, for example, mean diameter, average spacing, and top height. In some circumstances, however, a more detailed state description may be necessary.

In addition to the transition functions describing growth and mortality, most growth models also include several auxilliary relationships. These usually include equations to estimate the instantaneous change in the state variables caused by treatments (e. g. the change in basal area resulting from thinning a certain number of trees per hectare), and to estimate volumes of various products given the state.

Forecasting may be done by using integrated forms of the local transition function, or through numerical integration or accumulation. It is useful not to confuse a growth model with its computer implementation. Different implementations of the same model may be appropriate for different applications (long-term forecasting, silvicultural regime evaluation, updating of stand records).

For more on the state-space approach in growth modelling see Garcia (1979). Incidentally, these ideas can be applied to any dynamic system, and in particular, to models for forest management planning (Garcia 1984b).

Types of growth models

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Dynamic growth models can be classified according to the level of detail in the state description, as follows.

Stand-level growth models describe the state of the stand by a few variables representing stand-level aggregates such as basal area, mean diameter, volume per hectare, stems per hectare, average spacing, top height, etc. Sometimes, parameters of diameter and/or height distributions are also used, although more often these are estimated *a posteriori* as functions of the state variables. The transition function is usually given as a system of difference or differential equations for the rates of change of the state variables (growth and mortality). Graphical methods have also been used.

In most situations, this type of model is likely to be the most appropriate for management planning of forest plantations. Some examples are Beekhuis (1966), Alder (1980), and Garcia (1984a, 1988). Other work published between 1973 and 1976 is listed, with abstracts, in CAB (1977).

Tree position models, or distance-dependent individual tree growth models, use a much more detailed state description. This includes the location (co-ordinates) and diameter, and sometimes height and crown dimensions, of every tree in a sample plot. Growth and mortality probabilities for each tree are expressed as functions of their dimensions and of the relative position and dimensions of their neighbors. Representative examples include Newnham (1966, 1968), Van Laar (1969), Mitchell (1975), and

Tennent (1982). See also Dudek and Ek (1980).

These models can be useful as research tools to study practices affecting tree spacial relationships in ways that stand-level variables cannot describe satisfactorily; for example, thinning by rows or other systematic patterns, management of mixed species stands, or heavy selective pruning. They may also provide insights into stand dynamics that could contribute to the development of better stand models. Direct management use of these models is hampered by their high computational cost and by the very detailed inventory information that they would require. When used in practice, often a stand-level description is used for generating a fictitious sample of individual tree locations and sizes, which is then used as an input to the model. This is conceptually equivalent to a stand-level growth model, with a rather complicated transition function.

Distance-independent individual tree growth models describe the state through individual tree data, but without specifying tree locations. Strictly speaking, this class of models should include only those based on a list of the actual trees in a sample plot, with their dimensions, as in Goulding (1972). It is common, however, to include in this class models where the state is a size distribution (usually a diameter distribution) specified by a stand table (histogram) or by a fixed number of distribution quantiles (Clutter and Allison 1974, Alder 1979), although it can be argued that this is a stand-level description. Additional references are given in Dudek and Ek (1980).

These models occupy an intermediate position between the stand-level and the tree-position models in terms of state description detail, computational cost, and information requirements. This detail is needed for modelling uneven-aged stands. With reasonably homogeneous forest plantations, however, the additional detail may be largely redundant.

A potential difficulty with tree size distributions arises from the spatial correlation of tree sizes (Garcia 1984a). Over very short distances there is usually a negative correlation due to competition. Over longer distances, microsite similarity causes a positive correlation, decreasing with distance. This implies that a tree size distribution must vary with the area of land considered. In particular, the variance must vary with plot size, and distributions derived from sample plots are unlikely to apply to whole stands or compartments. Curiously, these considerations have generally been ignored by growth modellers although their importance has long been recognized in forest sampling. The practical significance of these effects for growth prediction is yet unknown. Until this is elucidated, however, it seems prudent to use tree size distributions with some care.

Estimation

Most growth model parameter estimation is done using linear or nonlinear regression. Several characteristics of growth modelling data may cause difficulties because of violation of statistical assumptions underlying regression techniques.

One problem often mentioned is the correlation between repeated measurements of permanent plots (Sullivan and Reynolds 1976, Ferguson and Leech 1978). This correlation arises because the value of a variable at the time of a measurement subsumes the values at previous measurements. The effect is more important for the development of static models. In dynamic models the dependent variables in the regressions are usually periodic increments, which are more nearly independent than the actual measurements. Some correlation is still present between successive increments, due mainly to measurement errors at the shared measurement points, and between increments for the same period in different plots, due to the action of similar weather conditions.

Another problem is the multiresponse nature of the models. That is, the models often consist of equations where the dependent variables may be correlated, and where parameters in different equations may be shared or may be functionally related. In this situation, fitting the equations one at a time may not be satisfactory, if at all possible. Hunter (1967) discusses the problem and some solutions in the context of chemical kinetics. Burkhart (1985) reviews approaches that have been tried in growth modelling. Bates and Watts (1985) have written recently on multiresponse estimation.

A third problem is the determination of increments or rates of change of the state variables from the data. Sometimes measurements have been taken evenly spaced in time. In this situation the computation of periodic increments to be used as dependent variables is straightforward. Often, however, measurements taken at various intervals are available, and approximations must be used. This is likely to be more of a problem with fast-growing species, where growth differences arising from varying dates of measurement are greater. In addition, the variability in increments due to year-to-year climatic fluctuations tends to be larger than in slow-growing stands where periodic increments average out these effects over several years. A related difficulty lies in the approximations sometimes needed when accumulating increments over a forecasting interval which is not an exact multiple of the increment periods.

Some statistical estimation methods not based on regression have been used. In a series of models for New Zealand radiata pine (Garcia 1979, 1984a, 1988), the differential equations specifying the transition function were augmented by random perturbation terms for estimation purposes, transforming them into stochastic differential equations. The problems associated with varying increment periods were then avoided by using directly the integrated form of the equations, at the same time accounting for most of the correlation effects through the stochastic structure of the model. Multiresponse was dealt with through simultaneous maximum likelihood estimation, using a general-purpose optimization procedure.

Summary

The state-space point of view can clarify the various approaches to growth modelling. In this view the behaviour of a time-varying system is described by a state that characterises the system at any point in time, and a transition function that specifies how the state changes over time.

A multidimensional state is required to adequately model forest stand growth. Growth models are commonly classified into three types that differ in the level of detail in the state description. In stand-level models the state consists of a small number of summary variables, for example basal area, stocking, and top height. Individual-tree distance-dependent models include in the state the size and location of every tree in a piece of land. Individual-tree distance-independent models use a state description based on a size (usually diameter) distribution.

The most appropriate type of model to use depends on the circumstances. The homogeneity of the stands and the kind of treatments to be analysed determine how detailed a state description needs to be. In addition, the state description also determines the quantity and quality of inventory data required for growth projections.

The development of growth models presents special statistical problems. An approach involving stochastic differential equations and maximum likelihood estimation has been developed and used successfully in New Zealand.

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